MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

TDA3231 – ALGORITHM DESIGN AND ANALYSIS

(All Sections/Groups)

7 March 2019
2.30pm – 4.30pm
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **4** pages with **4** questions only.
2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.

Question 1

(a) ***prefixAverages(X, n)*** below is an algorithm to compute the prefix averages of an array X of n integers.

```
Algorithm prefixAverages(X, n)
Input array X of n integers
Output array A of prefix averages of X
A ← new array of n integers
for i ← 0 to n − 1 do
        s ← X[0]
        for j ← 1 to i do
                s ← s + X[j]
        A[i] ← s / (i + 1)
return
```

(i) What is the big O time complexity of the algorithm? (1 mark)

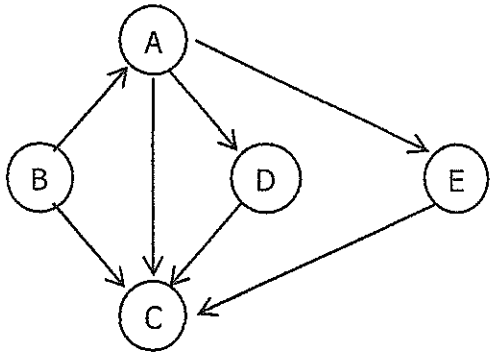
(ii) Show the steps involved in counting the big O time complexity for each statement of the algorithm. (2 marks)

(b) The algorithm ***prefixAverages(X, n)*** above can be improved in terms of time complexity.

(i) Write the improved version of the algorithm. (2 marks)

(ii) Show the steps involved in counting the big O time complexity for each statement of the algorithm. (2 marks)

(c) The following directed acyclic graph is provided. Draw the two possible topological orders for the graph. (3 marks)



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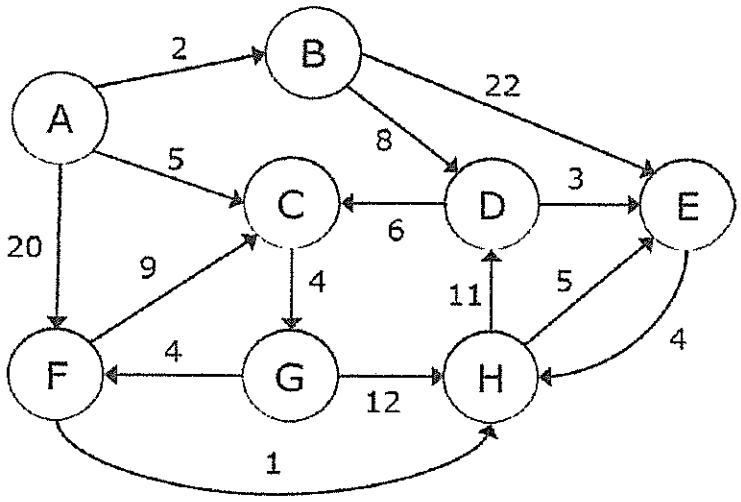
Question 2

(a) Consider the following directed, weighted graph:



(i) Step through Dijkstra's algorithm to calculate the single-source shortest paths from A to every other vertex. Show your steps in the table below. Cross out old values and write in new ones, from left to right within each cell, as the algorithm proceeds. (4 marks)

Vertex	Known	Distance	Path
A	Y	0	–
B	Y	2	A
C			
D			
E			
F			
G	Y	9	C
H	Y	~~17~~ 14	~~E~~ F

(ii) List the vertices in the order which you marked them known. (1 mark)

(iii) Indicate the lowest-cost path from node A to node G. (1 mark)

(b) Let n denote the number of items in an array to be sorted; and let b denote the maximum number of steps for sorting an array of one item (basis case, $n < 2$). Let $T(n)$ denote the running time of a recursive Merge Sort algorithm, which can be expressed in terms of a recurrence equation, as follows:

$$T(n) = \begin{cases} b & \text{if } n < 2 \\ 2T(n/2) + bn & \text{if } n \geq 2 \end{cases}$$

Prove by using iterative substitution that $T(n)$ is $O(n \log n)$. (4 marks)

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Question 3

(a) What is the difference between a Fractional Knapsack problem and a 0/1 Knapsack problem? (2 marks)

(b) Why doesn't Greedy Algorithm work for the 0-1 Knapsack Problem? Explain with an example. (3 marks)

(c) Solve the following 0/1 Knapsack problem using Dynamic Programming:

Given: A set S of n items, with each item i having
- b_i - a positive benefit
- w_i - a positive weight

Goal: Choose items with maximum total benefit but with weight at most W. An item must be taken as a whole (taking an item partially is not allowed).

Suppose that you have a knapsack that can carry maximum weight W = 11kg of load. The following items are available for your selection.

Item i	1	2	3	4	5
Weight w_i (kg)	1	2	5	6	7
Benefit b_i (RM)	1	6	18	22	28

Fill up the rows of the table below. The first and second rows (k = 0 and k = 1) have been filled up for you. What is the maximum benefit you can obtain? (5 marks)

Items	Weight (kg)											
	0	1	2	3	4	5	6	7	8	9	10	11
k = 0 {}	0	0	0	0	0	0	0	0	0	0	0	0
k = 1 {1}	0	1	1	1	1	1	1	1	1	1	1	1
k = 2 {1,2}												
k = 3 {1,2,3}												
k = 4 {1,2,3,4}												
k = 5 {1,2,3,4,5}												

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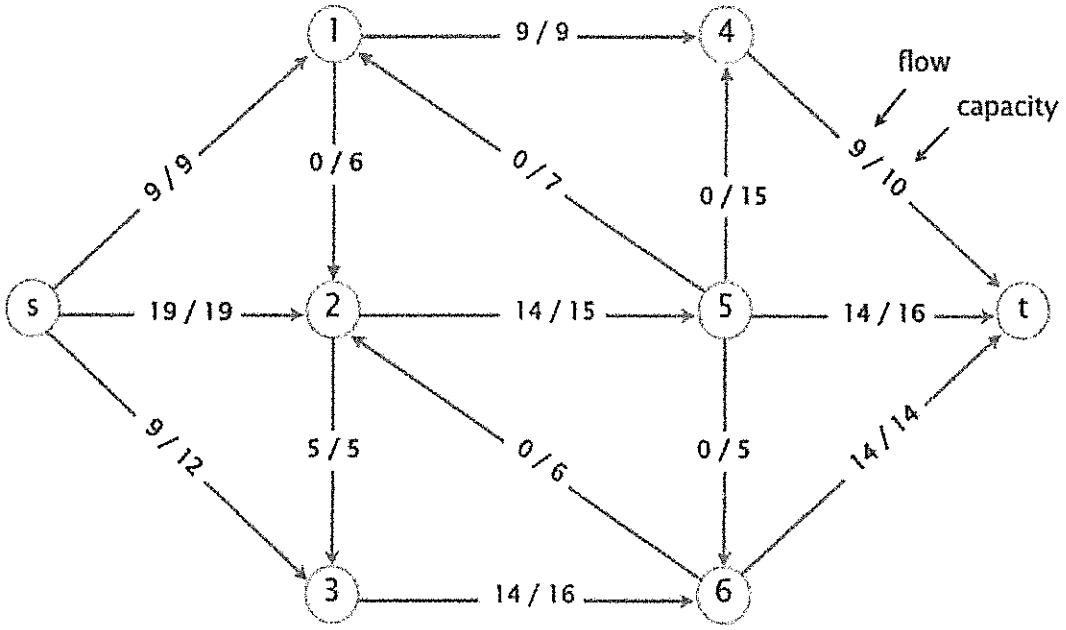
Question 4

(a) The following directed graph which represents a *st*-flow network (i.e., flow from source *s* to sink *t*) is provided:



(i) With the given flow and capacity values in the above network, what is the total value of the flow at the sink *t*? (1 mark)

(ii) Apply Ford-Fulkerson algorithm, from *s* to *t*, on the above network. List all the possible augmenting paths. Each path is represented as a sequence of vertices. (2 marks)

(iii) What is the value of the maximum flow? (2 marks)

(b) (i) What does it mean for a problem to be in NP-complete? (2 marks)

(ii) For the following three problems, state whether they are P, NP, NP-Complete, or any combination of two or three of them. (3 marks)

Problem 1 - Finding the maximum value in an array.

Problem 2 - Finding a cycle that visits each vertex in a graph exactly once.

Problem 3 - Finding a cycle that visits each edge in a graph exactly once.

End of Paper